Abstract—This paper proposes a procedure based on
time-difference of arrival measurements to localize a blind
node in an asynchronous network where a set of nodes with
known position is present. The proposed method computes
the time-difference of arrival of each transmitted signal to
any pair of receiving nodes in order to get rid of the
unknown transmission time. A range-based localization
procedure is implemented: first a least-squares estimator
is used to compute a set of pseudo-ranges involving the
blind node; then an iterative least-squares method is used
to localize the blind node. The effectiveness of the proposed
scheme is illustrated via simulations.

I. INTRODUCTION

Several Wireless Sensor Network (WSN) scenarios
involve mixtures of fixed and mobile nodes, where only a
subset of the nodes are placed in known positions. In this
context, it is desirable to endow the WSN system with
the capability of automatically locating the remaining
(fixed or mobile) nodes placed in unknown positions.
One obvious option is to equip the mobile node(s)
with additional positioning hardware, e.g. GPS receiver.
However, this approach might not be viable due to
constraints in the size, power and/or cost of the mobile
device, not to mention the case when GPS signals are
not available in the operating environment (e.g. indoors,
tunnels, urban canyons). This motivates the interest
for radio positioning techniques that “opportunistically”
leverage the legacy radio signals used by the WSN for
communication, without requiring modifications to the
underlying protocols.

We explore here an approach that relies on time mea-
surements: the key variables that drive the localization
procedure are the reference signal or packet time of
arrivals at the generic receiving node with respect to
its local clock. We consider an asynchronous WSN
system where nodes are not synchronized and each node
clock is affected by an unknown temporal offset (bias)
with respect to an ideal reference. Also, we do not
require any particular control of or information about
the transmitting time, therefore we are able to cope with
asynchronous transmissions. We formulate and solve in
closed-form the ranging estimation problem in presence
of asynchronous transmissions.

Likewise some recent pioneering studies [1], [2],
[3], [4], [5], [6], [7] we also rely on time-difference
of arrival (TDOA) measurements. However, the system
model and the problem formulation proposed here differ
from those investigated therein. For instance, the method
considered in the previous work [7] was based on TDOA
computed at cooperative nodes acting as receivers based
on available pairs of time-of-arrival measurements from
different transmitters. In this way, we could get rid
of the unknown bias of the receivers in the problem
formulation. Conversely, the method proposed here is
based on TDOA between signals propagating from any
transmitter to any pair of receiving nodes in order to
get rid of the unknown transmission time. We consider
a symmetric and homogeneous scenario where transmis-
sions and receptions, being from/to the blind node or
from/to the anchor nodes, all contribute to collectively
determine the blind node position. This scenario fits best
WSN systems, where node transmissions tend to be bi-
directional — at least up to the MAC layer, due to

ACKnowledgments.

The remainder of the paper is organized as follows:
next section is devoted to problem formulation while
Section III addresses the estimator design. Finally, Sec-
tion IV assesses the performance of the proposed scheme
and contains some concluding remarks.

II. PROBLEM FORMULATION

The reference scenario involves anchors, namely co-
operative nodes of known position capable of acting
as receivers and transmitters and a blind node, also
acting as receiver and transmitter, whose position must
be determined. For the sake of simplicity, we refer here
to a centralized architecture, with a central server gath-
ering all data from the nodes (including the blind node)
and performing the computation. However, it should be possible to adapt the problem formulation to work in (semi-)distributed communication schemes.

The determination of the blind node position involves two distinct stages: pseudo-range estimation and position estimation. In the first stage, timing measurements between participating nodes and positions of anchors serve as input for the estimation of the pseudo-ranges between the blind node and the anchors (actually, a subset of the nodes, located within the reception range of the blind node). In the second stage, the pseudo-range estimates are used as input for determining the position of the blind node. We restrict our attention to planar (2D) localization where the minimum number of anchors required to solve the positioning problem is three.

More specifically, we assume that there are \( M - 1 \) anchors (\( M \) nodes, including the blind node) in a fully-meshed topology: in other words, we assume that every packet transmitted by a node is correctly received by all the other \( M - 1 \) nodes. Moreover, we denote by \( \mathcal{A} = \{1, \ldots, M\} \) the set of integers indexing the nodes; the blind node is indexed by 1. Considering the transmission of a reference signal or packet from transmitting node \( i \) to receiving node \( k \), we introduce the following quantities:

- \( t_i \) is the (unknown) transmission time of the reference signal from transmitter \( i \) according to the absolute reference clock.
- \( r^k_i \) is the measured time when the reference signal transmitted by node \( i \) was received at node \( k \).
- each of the \( M \) nodes will be characterized by a clock bias: we denote by \( b_k \) the clock bias of the \( k \)th node.

Without loss of generality we may take the clock of all receivers. This is straightforward in practice since wireless protocols embed a variety of well-known patterns such as training sequences, preambles, delimiters and alike.

**Equations.** The formulation of the estimation problem (first stage) involves equations of the type

\[
r^k_i + b_k = t_i + \frac{d_{ik}}{c} + \epsilon_{ik}, \quad i, k \in A, i \neq k
\]

where \( c \) is the speed of light and \( \epsilon_{ik} \) denotes a measurement error with variance \( \sigma^2_{ik} \). Notice that the variance of the generic measurement error depends on several factors as, for instance, the characteristics of the transmitted signal, namely bandwidth and duration, and the signal-to-noise ratio (SNR) at the receiver.

Assuming \( M \geq 3 \), such equations can be used to compute the TDOA statistics, i.e.,

\[
\delta r^k_{i,h} = r^k_i - r^h_i = b_h - b_k + \frac{d_{ih} - d_{ik}}{c} + \epsilon_{ih}, \quad \forall i, h, k \in \mathcal{A}, i \neq k, i \neq h, h \neq h.
\]

### III. Resolution Approach

The unknown deterministic parameters can be grouped to form the following vector

\[
x = [b_2 \cdots b_M \ d_1 \cdots d_{1,M}]^T
\]

with \( T \) denoting the transpose operator. Similarly, we can construct a vector of observables \( y \) as

\[
y = [y_1^T \ y_2^T \ y_3^T]^T
\]

with

\[
y_1 = [\delta r^1_2, \ldots, \delta r^1_M, \ldots, \delta r^{1,2}_M, \ldots, \delta r^{1,M-1}_M]^T
\]

\[
y_2 = [\delta r^{2,3}_1, \ldots, \delta r^{2,4}_1, \ldots, \delta r^{3,4}_1, \ldots, \delta r^{3,M-1}_1, \ldots, \delta r^{4,M-1}_1, \ldots, \delta r^{M-1,M}_1]^T
\]

and

\[
y_3 = [\delta r^{2,3}_2, \ldots, \delta r^{2,4}_2, \ldots, \delta r^{3,4}_2, \ldots, \delta r^{3,4}_2, \ldots, \delta r^{M-1,M}_2, \ldots, \delta r^{M-1,M}_2]^T
\]

Notice that \( y_1 \) contains the measurements involving the blind node as a receiver, \( y_2 \) those involving the blind node as a transmitter, and \( y_3 \) collects the remaining measurements between the anchor nodes. It follows that

\[
y = Hx + w
\]

where \( w \) is the vector of the noise terms; its \( i \)th entry is the noise term of the \( i \)th component of \( y \). Notice that the number of rows of \( H \) is \( M(M-1)(M-2)/2 \) and the number of columns is equal to \( 2(M-1) \); it follows that for \( M \geq 4 \) the matrix is a tall one.

The problem of estimating \( x \) can be solved resorting to the least-squares (LS) method; it follows that

\[
\hat{x} = \arg \min_x \| y - Hx \|^2
\]
It is well-known that the solution to problem (3) is unique only if the tall matrix $H$ is a full-column rank one. However, the variables in $x$ cannot be univocally identified since they always appear in "pairwise differences" in (2). In fact, if $\hat{x}_0$ is a solution of the LS problem, it is apparent that

$$\hat{x} = \hat{x}_0 + \mu \left[ -1^{T}_{M-1} \frac{1}{c} 1^{T}_{M-1} \right]$$

with $\mu \in \mathbb{R}$ and $1_n$ an $n$-dimensional vector of ones, is a solution too. Thus, it turns out that, for the problem at hand, the matrix $H$ is not a full-column-rank matrix; as a matter of fact, it is possible to show that the matrix is rank deficient by just one and, hence, the solution is unique up to a known vector multiplied by an unknown constant, namely (4) describes the overall set of solutions of the problem at hand. The unknown $\mu$ can be removed in the position estimation stage by resorting to a plain iterated LS algorithm based on three or more pseudo-ranges [8]. Herein, we compute $\hat{x}_0$ as

$$\hat{x}_0 = H^\dagger y$$

where $H^\dagger$ is the Moore-Penrose generalized inverse of $H$ and $\hat{x}_0$, given by (5), is the unique minimum norm solution of the LS problem (3) [9].

IV. SIMULATION RESULTS

In this section, we present Monte Carlo simulation results in order to validate the correctness of the proposed approach. We consider a squared area of $3 \times 3$ km$^2$ with $M$ nodes. The position of the anchors is shown in Figure 1: simulation results obtained assuming $M$ anchors refer to those anchors labeled as $2, \ldots, M$. At each of the 1000 trials of the simulation, the position of the blind node is generated randomly within the squared area of Figure 1. The propagation effects over all links are simulated by neglecting possible multiple paths and considering a path loss exponent $\alpha = 2$. Moreover, the SNR at the receiver is modeled as

$$SNR = \frac{E_s/N_0}{d^\alpha}$$

where the energy contrast $E_s/N_0$ is equal to 20 dB at 1 km from the transmitter and $d$ is the length of the link (in km). In particular, we model distinct timing errors as independent, zero-mean Gaussian random variables (rvs). In other words, we assume that $\epsilon_{ik}$ and $\epsilon_{ih}$ are independent rvs whether $k \neq h$. Finally, the variances of timing errors have been set according to the modified Cramer-Rao lower bound [10]. In particular, we set the variance multiplying the modified Cramer-Rao lower bound by a factor $\gamma \geq 1$. To this end, we assume root-raised-cosine-rolloff pulses with a normalized mean square bandwidth $\beta^2 \approx 0.1$, a symbol duration $T$ with $1/T = 2$ MHz, and an estimation interval of 64 symbols.

Figures 2, 3, 4, and 5 report the empirical cumulative distribution function (ECDF) of the module of the positioning error of the blind node for $M = 4, 6, 8$, and 10, respectively, $\gamma$ as a parameter. Remarkably, for reasonable values of $\gamma (1 \leq \gamma \leq 10)$ and $M (M \geq 6)$, the median localization error of the proposed method is less than ten meters. A definite evaluation of the proposed scheme is part of the ongoing research work also considering a more realistic propagation environment where shadowing and fading are taken into account.
Fig. 3. Simulation results for a scenario assuming $M = 6$, $\gamma$ as a parameter ($\gamma = 1, 5, 10, 15, 20, 25, 30$).

Fig. 4. Simulation results for a scenario assuming $M = 8$, $\gamma$ as a parameter ($\gamma = 1, 5, 10, 15, 20, 25, 30$).

Fig. 5. Simulation results for a scenario assuming $M = 10$, $\gamma$ as a parameter ($\gamma = 1, 5, 10, 15, 20, 25, 30$).

REFERENCES